Media Trading Groups and Short Selling Manipulation—Are Media Groups Efficiency Enhancing or Reducing?

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TWIPS, Cornell

April 9, 2021

Introduction

- Meme stock craze: bear raids or day traders revolution?
  - *GameStop and YOLO: "You Only Live Once"*
- Social media, coordination cost, and media trading groups
- Media groups reinforce new reality for hedge funds.
  - New internal rules, avoid crowded trades or small stocks
  - The market goes dark, i.e., trading off the exchange.
- Media trading groups and market efficiency.
  - Is it necessary to regulate media groups?

Result preview

- We construct a competitive equilibrium model with strategic trading and media trading groups.
- Chatroom traders form a coalition and select a coordinator to do the media chat.
- Media trading groups can discipline short sellers.
- Various phenomena arise, including short squeeze, forced liquidation, and precautionary savings.
- Media groups can be both efficiency enhancing and reducing.

Related Literature

- Speculation and manipulation.
  - Trade based manipulation (Jarrow, 1992; Allen and Gale, 1992; Allen and Gorton, 1992; Chakraborty and Ylmaz, 2004a,b; Goldstein and Guembel, 2008; Williams and Skrzypacz, 2020).
  - information-based manipulation (Vila, 1989; Kumar and Seppi, 1992; Benabou and Laroque, 1992; Gerard and Nanda, 1993; Bagnoli and Lipman, 1996; Van Bommel, 2003; Chakraborty and Ylmaz, 2008)
- Bubble creation via short sale constraints under heterogenous beliefs (Santos and Woodford, 1997; Scheinkman and Xiong, 2003; Hong et al., 2006; Scheinkman, 2013).
  - very little has been written on the equilibrium foundations and implications of market corners and short squeezes (Kyle, 1984; Villa, 1987; Pirrong, 1993; Cooper and Donaldson, 1998; Allen et al., 2006).
- Theory of labor union (Taschereau-Dumouchel, 2020)

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The Model

- A two period model. Trading occurs at time 0, all payoffs at time 1.
- Two types of traders. A continuum of small traders of measure $I > 0$, and a large trader (hedge fund)
- Given is $(\Omega, \mathcal{F}, \mathbb{P})$. Two assets. A money market account and a risky stock. The stock liquidation value $\xi : \Omega \to \mathbb{R}_+$. The stock has a supply $N > 0$. Market is frictionless except for various short sale restrictions.
- **Momentum trading:** the stock value at time 1 is given by $S = \xi h(\kappa, z)$, where $h : \mathcal{K} \times \mathcal{Z} \to \mathbb{R}_+$ with $h(0,0) = 1$.
- A Competitive equilibrium corresponds to $s(\kappa, z) : \mathcal{K} \times \mathcal{Z} \to \mathbb{R}_+$ by equating demand with supply.

Timeline

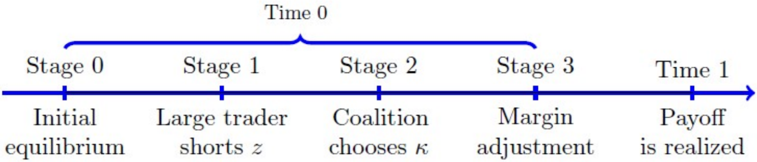

Figure 1: Timeline

Small Traders

1. Consider a complete market.
   - The stock evolution must be binomial, i.e., $\Omega = \{H, L\}$, $\mathbb{P}(H) = p$.
2. The ith small trader is risk neutral with $\mathbb{P}_i$ s.t. $\mathbb{P}_i(H) = \theta_i$.
   - $\theta_i \in [0, 1]$ is drawn with density $f(\theta_i) > 0$ and CDF $F(\theta_i)$.
3. **Assumption 1:** i)Trader i's trade size $d_i \in [0, 1], \forall i \in I$;
   ii) Trader i's initial wealth $W_i \geq \sup_{\kappa \in \mathcal{K}} h(\kappa, 0), \forall i \in I$.
4. The ith small trader's problem solves
   $$d_i^*(s) = \arg \max_{d_i \in [0,1]} d_i \mathbb{E}_i[h(\kappa, z)\xi] - d_i s + W_i. \quad s.t.\ d_i s \leq W_i.$$
5. $d_i^* = 1\left(\theta_i \geq \frac{s}{h(\kappa,z)}\right)$ and $\int_{i \in I} d_i^*(s) = I\left[1 - F\left(\frac{s}{h(\kappa,z)}\right)\right]$.

Large Trader

- He is risk neutral with $\mathbb{P}_L$ s.t. $\mathbb{P}_L(H) = \theta_L$ and initial wealth W_L.
- **Assumption 2:** $I\Big(1 - F\big(h_1(\bar{\kappa})\theta_L\big)\Big) > N$ where $h(\kappa, z) := h_1(\kappa)h_2(z)$ with $h_1(0) = h_2(0) = 1$.
    - This implies that $z^* < 0$.
- The large trader's problem solves

$$z^* = \arg\max_{z \in \mathcal{Z}} z\Big(E_{\mathbb{P}_L}[\xi h(\kappa, z)] - s(0, z)\Big) + W_L$$
$$s.t. \ \left|zs(\kappa, z)\right| m \leq W_L.$$

  - m: marginal mutiplier, which makes him behave as if he is risk averse. Currently, $m = 1.5$.
  - Note that the belief κ enters the objective.

The Equilibrium Stock Price

Lemma 1

Given the large trader's short position $z < 0$ and the coalition strength κ, the equilibrium stock price $s(\kappa, z)$ is

$$s(\kappa, z) = h(\kappa, z)\theta(z) \tag{1}$$

where $\theta(z) := G\left(\frac{I-N+z}{I}\right)$ and $G(\cdot) = F^{-1}(\cdot)$.

- This follows from market clearing: $N + |z| = I\left[1 - F\left(\frac{s(\kappa,z)}{h(\kappa,z)}\right)\right]$.
- Key properties:
  1. $\theta(z)$ is the marginal investor's belief, and defines the martingale measure.
  2. $\frac{d\theta(z)}{dz} > 0$, $\frac{ds(\kappa,z)}{d|z|} < 0$, and $\frac{ds(\kappa,z)}{\partial\kappa} > 0$.

Media Group

- GameStop frenzy: Nio, AMC, Plug Power, Blackberry,...
- Who are the social medium enabled day traders?
- How does media trading groups work? *Chatroom traders gather in online communities such as TikTok, Twitter, Reddit, and the messaging platform Discord. They piggyback on each other's ideas and trades, and they coordinate via these popular social media, thereby affecting the stock price.*
- Similar to a labor union (Taschereau-Dumouchel, 2020), day traders partly delegates decision rights to a coordinator.

Market Structure

Timing	Time 0					Time 1
	Stage 0	Stage 1		Stage 2	Stage 3	
Actions & beliefs	No coalition No short-selling	Large trader chooses z	anticipates a coalition	Coalition chooses κ	Forced Liquidation	Stock payoff is realized
No media	YES	YES	NO	NO	NO	YES
Unexpected media	YES	YES	NO	YES	Possible	YES
Expected media	YES	YES	YES	YES	NO	YES
Unexpected No media	YES	YES	YES	NO	NO	YES
Incomplete Information	YES	YES	w.p. λ	If coalition is formed	Possible	YES

Figure 2: Market Structure

Market Structure—An Example

- Consider the "Unexpected No media" market structure.
- In stage 0, no short selling and no media group, and the price is $s_0 = s(0, 0)$.
- In stage 1, the large trader shorts the stock, believing $\kappa = 0$. The optimal short position is z_u^*, and $s_1 = s(0, z_u^*)$.
- In stage 2, a media group arises, and choose $\kappa^* = \kappa_u^*$ and $s_2 = s(\kappa_u^*, z_u^*)$.
- In stage 3, a margin call might arise since $s_2 > s_1$. When the budget constraint is binding, the short position is revised.

$$W_L = \underbrace{|\check{z}_u| s(\kappa_u^*, \check{z}_u)| m}_{\text{new margin}} + \underbrace{|\check{z}_u - z_u^*| s(\kappa_u^*, \check{z}_u)}_{\text{liquidation cost}}.$$

- Note that $z_u^* < \check{z}_u \leq 0$.

Equilibrium Analysis

- We use backward induction.
- In stage 3, if forced liquidation, $\check{z} \in (z, 0]$; otherwise, $\check{z} = z$.
- In stage 2, if there is no media group, $\kappa^* = 0$; if media group arises, a ccordinator solves

$$
\begin{aligned}
(d^*, \kappa^*) &= \underset{(d_i,\kappa)\in[0,1]\times[0,\bar{\kappa}]}{\arg\max} \; d_i\big(\mathbb{E}_i[\xi h(\kappa, z)] - s(\kappa, z)\big) + W_i \\
&= \underset{(d_i,\kappa)\in[0,1]\times[0,\bar{\kappa}]}{\arg\max} \; d_i h(\kappa, z)\big(\theta_i - \theta(z)\big) + W_i
\end{aligned}
$$

$$
s.t. \quad d_i s(\kappa, z) \le W_i
$$

  - $d_i^* = 1\big(\theta_i \ge \theta(z)\big)$, and $\kappa^* = \bar{\kappa}$.

Equilibrium Analysis cont'd

- In stage 1, the large trader's problem solves

$$z^* = \arg\max_{z \in \mathcal{Z}} z\big(\theta_L h(\kappa, z) - h(0, z)\theta(z)\big) + W_L \tag{2}$$

$$s.t. \quad |z| s(\kappa, z) m \leq W_L. \tag{3}$$

- Assume a product form for the momentum effect, i.e., $h(\kappa, z) = h_1(\kappa) h_2(z)$.
- **Assumption 3:** (Concavity of the Large Trader's Objective Function)
(i) $|z| h_2(z)\theta(z)$ is weakly decreasing in $z \in \mathcal{Z}$, and
(ii) $z h_2(z)\big(h_1(\kappa)\theta_L - \theta(z)\big)$ is concave in $z \in \mathcal{Z}$, $\forall \kappa \in \mathcal{K}$.
- **Example 1.** Take $h_2(z) = 1, \forall z \in \mathcal{Z}$ and $\theta_i \in \mathrm{Uniform}([0, 1])$. Then, Assumption 1-3 hold.

Eqlm Analysis cont'd

- Define $\widehat{W_L}(\kappa, z) := m|z|s(\kappa, z) = m|z|h_1(\kappa)h_2(z)\theta(z)$.
- Given the belief κ, define by $z^{\dagger}(\kappa)$ the unconstrained optimizer to Eq. (2), and by $z^{\intercal}(\kappa)$ when Eq. (3) is binding.

Proposition 1

Suppose Assumptions 1, 2, and 3 hold. Then: (i) the optimal media visibility is $\kappa^* = \bar{\kappa}$, whenever a coalition is formed; otherwise, $\kappa^* = 0$, and (ii) given that the large trader's believes the coalition's choice is κ, the optimal short-selling volume z^* is

$$z^*(\kappa) = \begin{cases} z^{\dagger}(\kappa), & \textit{if} \quad W_L \geq \widehat{W_L}(\kappa, z^{\dagger}(\kappa)); \\ z^{\intercal}(\kappa), & \textit{otherwise} \end{cases}$$

No Media

Proposition 2

Under no media market structure, $z_n^* = z^\dagger(0) \vee z^\intercal(0)$.

- **Assumption 4:** $V_0(z) := \frac{-zh_2(z)}{If((I-N+z)/I)[h_2(z)+zh_2'(z)]}$ is decreasing in z.

Proposition 3

(Monotonicity) The unconstrained optimum, $z^\dagger(\kappa)$*, and the maximum affordable short selling volume,* $z^\intercal(\kappa)$*, are both increasing in* κ.

Corollary 2

(Short Selling Incentives) When the large trader anticipates no coalition, his incentive for short selling is strongest.

No Media (cont'd)

Define $c(z_n^*) = \frac{\int_{\theta(0)}^{1}\left(\theta - \theta(0)\right) dF(\theta)}{\int_{\theta(z_n^*)}^{1}\left(\theta - \theta(z_n^*)\right) dF(\theta)}$ and $\tilde{\theta}(z_n^*) := \theta(0) + \frac{h_2(z_n^*)(\theta(0) - \theta(z_n^*))}{1 - h_2(z_n^*)}$.

Proposition 4

(Small Trader Surplus)
i) When $h_2(z_n^*) \geq c(z_n^*)$, $\Delta CS \geq 0$, so the small traders are better off. In contrast, when $h_2(z_n^*) < c(z_n^*)$, $\Delta CS < 0$, the small traders are worse off.
ii) Choose z_n^* such that $h_2(z_n^*) = c(z_n^*)$, so that $\Delta CS = 0$. For each individual trader, the most optimistic $\theta_i \in \left(\tilde{\theta}(z_n^*), 1\right]$ are worse off after short selling, while those with beliefs $\theta_i \in \left[\theta(z_n^*), \tilde{\theta}(z_n^*)\right]$ are better off.

Unexpected Media

- Recall that $W_L = m|\check{z}_u|s(\bar{\kappa}, \check{z}_u)| + |\check{z}_u - z_u^*|s(\bar{\kappa}, \check{z}_u)$.

Lemma 3

Under the unexpected media market structure, the eqlm strategies are
i) in stage 1, $z_u^* = z_n^* = z^\dagger(0) \vee z^\intercal(0)$,
ii) in stage 2, $\kappa^* = \bar{\kappa}$,
iii) in stage 3, if $W_L < \widehat{W}_L(\bar{\kappa}, z_u^*)$, *then a forced liquidation happens and the short position is revised to* $\check{z}_u$; *otherwise, the short position remains unchanged at* z_u^*.

- Key features. i) Over Short Selling. ii) Short Squeezes. iii) Forced Liquidations.

Expected Media

Lemma 4

Under the expected media market structure, the equilibrium strategies are
i) in stage 1, the large trader's optimal short position is

$$z_e^* = \begin{cases} z^\dagger(\bar{\kappa}), & \textit{if} \quad W_L \geq \widehat{W_L}(\bar{\kappa}, z^\dagger(\bar{\kappa})) \\ z^\intercal(\bar{\kappa}), & \textit{otherwise} \end{cases}$$

ii) in stage 2, $\kappa^* = \bar{\kappa}$*, and*
iii) in stage 3, the large trader posts more margin and there is no forced liquidation.

- Key features: i) Anticipated margin call; ii) No forced liquidation; iii) Precautionary savings.

Unexpected no media

Lemma 5

Under the unexpected no media market structure, the equilibrium strategies are
i) in stage 1, the large trader's optimal short position is

$$z^*_{un} = z^*_e = z^\dagger(\bar{\kappa}) \vee z^\intercal(\bar{\kappa}),$$

ii) in stage 2, $\kappa^* = 0$*, and*
iii) in stage 3, there is no margin call.

- Key features. Conservative short selling.

Market Structure

Market Structure	Equilibrium Outcome				
	Shorting	Ex-post Optimality	Margin Call	Short Squeeze	Small Traders' Surplus
No media	Aggressive	YES	NO	NO	Lowest
Unexpected media	Aggressive	Over Shorting	YES	YES	Highest
Expected media	Conservative	YES	YES	NO	High
Unexpected No Media	Conservative	Under Shorting	NO	NO	Low

Figure 3: Market Structure and Equilibrium Outcome

Incomplete Information Game

- The idea: sophisticated short sellers consistently estimates the probability of a media group, say, $\lambda \in (0,1)$.
- The modeling difference. In stage 1, when shorting the stock, the large trader believes an uncertain media group. In stage 2, a media group is formed w.p. λ.
- How does it affect the large trader's incentive for shorting?
    - Forced liquidations never occur.

$$
\begin{aligned}
z^* &= \arg\max_{z \in \mathcal{Z}} z\Big(\mathbb{E}^{\lambda}_{\mathbb{P}_L}[S] - s(0,z)\Big) + W_L \\
&= \arg\max_{z \in \mathcal{Z}} z\Big(\big[\lambda h(\bar{\kappa}, z) + (1-\lambda) h(0,z)\big]\theta_L - s(0,z)\Big) + W_L \\
&= z h_2(z)\Big(\big[\lambda h_1(\bar{\kappa}) + 1 - \lambda\big]\theta_L - \underline{\theta}(z)\Big) + W_L \\
s.t.\ \ & m|z| s(\bar{\kappa}, z) \leq W_L
\end{aligned}
$$

Incomplete Information Game (cont'd)

- Forced liquidations might occur.
  1. Case 1. W.p. $1-\lambda$, no media group. The profit is $z\Big(\mathbb{E}_{\mathbb{P}_L}\left[\xi h(0,z)\right] - s(0,z)\Big) + W_L$.
  2. Case 2. W.p. λ, a media group is formed. The profit is

$$\underbrace{\check{z}\mathbb{E}_{\mathbb{P}_L}[\xi h(\bar{\kappa},\check{z})]}_{\text{Total Liability}} \underbrace{-zs(0,z)}_{\text{Initial Gains}} - \underbrace{s(\bar{\kappa},\check{z})(\check{z}-z)}_{\text{Liquidation Cost}} + W_L$$

- Denote $\mathbb{E}[U_L^1] = V_1(z) + V_2(z)$, where

$$V_1(z) = zh_2(z)\Big(\big[\lambda h_1(\bar{\kappa}) + 1 - \lambda\big]\theta_L - \theta(z)\Big) + W_L$$

and

$$V_2(z) := \lambda\Big(\big[\check{z}h(\bar{\kappa},\check{z}) - zh(\bar{\kappa},z)\big]\theta_L - \lambda s(\bar{\kappa},\check{z})(\check{z}-z)\Big)1(z < z^{\mathsf{T}}(\bar{\kappa}))$$

Equilibrium Construction

- **Assumption 5:** $V_2(z)$ is (weakly) concave in z, $\forall z \in \mathcal{Z}$.

Proposition 5

(Incomplete Information Equilibrium) Fix $\lambda \in (0,1)$. *Suppose Assumptions 1, 2, 3 and 5 hold. Assume* $\frac{d\check{z}}{d\kappa} \geq \sup_{(\kappa,z)\in[0,\bar{\kappa}]\times[z^{\intercal}(0),z^{\intercal}(\bar{\kappa})]} \left\{ -\frac{h_1'(\kappa)}{h_1(\kappa)}\frac{h_2(z)}{h_2'(z)} \right\}$.

i) In stage 1, the large trader's optimal short position is:

$$z^* = \begin{cases} z^{\dagger}(\lambda), & \text{if } W_L \geq \widehat{W}_L\left(\bar{\kappa}, z^{\dagger}(\lambda)\right); \\ \in \left(z^{\dagger}(\lambda), z^{\intercal}(\bar{\kappa})\right), & \text{if } W_L < \widehat{W}_L\left(\bar{\kappa}, z^{\dagger}(\lambda)\right) \;\&\; V_1'(z^{\intercal}(\bar{\kappa})) + V_2'\left(z^{\intercal}(\bar{\kappa})_-\right) < 0; \\ z^{\intercal}(\bar{\kappa}), & \text{if } W_L < \widehat{W}_L\left(\bar{\kappa}, z^{\dagger}(\lambda)\right) \;\&\; V_1'(z^{\intercal}(\bar{\kappa})) + V_2'\left(z^{\intercal}(\bar{\kappa})_-\right) \geq 0. \end{cases}$$

ii) In stage 2, the media group's optimal choice is:

$$\kappa^* = \begin{cases} \bar{\kappa}, & \text{if a media group exists} \\ 0, & \text{else.} \end{cases}$$

iii) In stage 3, a margin call only occurs when a media group emerges, which leads to a forced liquidation when $W_L < \widehat{W}_L\left(\bar{\kappa}, z^{\dagger}(\lambda)\right)$ *and* $z^* < z^{\intercal}(\bar{\kappa})$. *The newly revised short position* $\check{z}(z^*)$ *is such that* $|\check{z}(z^*)| < |z^*|$.

Market Structure

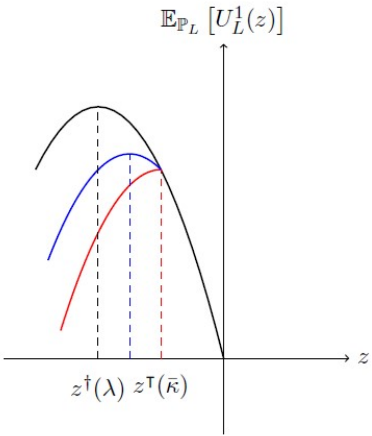

Figure 4: Equilibrium Structure

Media Group and Market Discipline

- Strong deterrence. A forced liquidation is preempted by a reduced short position and a forced liquidation never occurs, i.e., $W_L < \widehat{W_L}\left(\bar{\kappa}, z^{\dagger}(\lambda)\right)$ & $V_1'(z^{\intercal}(\bar{\kappa})) + V_2'\left(z^{\intercal}(\bar{\kappa})_-\right) \geq 0$.
- Weak deterrence. Here, the large trader willingly tolerates some forced liquidations to gain large profits from shorting when no media group arises, i.e., $W_L < \widehat{W_L}\left(\bar{\kappa}, z^{\dagger}(\lambda)\right)$ & $V_1'(z^{\intercal}(\bar{\kappa})) + V_2'\left(z^{\intercal}(\bar{\kappa})_-\right) < 0$.
- No deterrence. If $W_L \geq \widehat{W_L}\left(\bar{\kappa}, z^{\dagger}(\lambda)\right)$. Here, the large trader has sufficient wealth so that margin calls have no affect on her actions.

Proposition 6

(Beliefs and Deterrence) Fix $W_L \leq \widehat{W_L}(\bar{\kappa}, z^{\dagger}(1))$ *so that* $z^{\dagger}(1) < z^{\intercal}(\bar{\kappa})$*. There exists a belief* $\bar{\lambda} \in (0, 1]$ *such that, weak deterrence results if* $\lambda \in (0, \bar{\lambda})$*, and strong deterrence results if* $\lambda \geq \bar{\lambda}$*.*

Allocative Efficiency

- Momentum trading can last as long as 24-36 months. Potentially large deviation between price and fundamental.
- Recall p actual p.m. and $\theta(z)$ market martingale measure.
- Efficiency loss function $L(\mathbb{P}, \mathbb{Q}) = L(p, \theta(z)) := -\big|p - \theta(z)\big|$.

Proposition 7

(Allocative Efficiency)
Media groups decrease allocative efficiency when $p < \theta_L < \theta(0)$, and they increase allocative efficiency when $p \geq \theta(0) > \theta_L$.

- Policy implication. Based on proposition 7, we believe that little if any additional regulation of media group coalition formation is needed.

Conclusion

- This paper studies how chatroom traders unionized via social media platforms influence the stock price through momentum trading.
- The economic consequences of this coalition formation is analyzed in a quasi-competitive equilibrium framework with strategic trading.
- We show that media groups can discipline the large trader's incentive to short sell, but it can be either efficiency increasing or decreasing.
- Robust to extensions including incomplete markets, media visibility costs and liquidity costs.