Endogenous Liquidity Risk and Dealer Market Structure

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Abstract

This paper derives a liquidity cost process in a non-cooperative cost competition game among market makers and discusses its implication for the structure of a dealer market. The main result shows that there does not exist an equilbrium supporting both multiple market makers earning strictly positive profits and a well-behaved liquidity cost process (i.e. strictly increasing and convex) as documented in the price impact literature. Bertrand price competition arises as an equilibrium phenoemon, which naturally leads to the dominance of a cost-efficient market maker in the dealer market.

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1 Introduction

Asset market equilibrium theory with liquidity risk hinges on the structure and the properties of the liquidity cost process (Cetin et al., 2004; Cetin and Rogers, 2007; Jarrow, 2018a, 2019), which is generally assumed to be well behaved (strictly increasing and convex). Although these properties are intuitive (Jarrow, 2018a) and consistent with evidence from the market microstructure literature (Madhavan, 2000; Biais et al., 2005), it remains an open question as to how this well-behaved liquidity cost process arises in equilibrium. The purpose of this paper is to answer this question.

This paper derives a liquidity cost process in a non-cooperative cost competition game between potential market makers and we also discuss the implications for the structure of a dealer market. The main result shows that there does not exist an equilibrium supporting both multiple market makers earning strictly positive profits and a well-behaved supply curve (i.e. strictly increasing and convex) as documented in the price impact literature. This negates the possibility of non-trivial competitive Nash equilibrium in the Cournot game sense. Hence, Bertrand price competition is the equilibrium phenomenon, which naturally leads to the dominance of cost-efficient market makers in the dealer market.

Surprisingly, the intuition behind this result is straightforward. Consider the setting of Jarrow (2018a) with two identical market makers. Suppose there exists a non-trivial equilibrium (i.e., other than the one characterized by the zero-profit condition) supporting a strictly increasing and convex supply curve. By symmetry and convexity of the supply curve, efficiency requires investors to trade with both market makers. Then, the market makers have an incentive to deviate downward to reduce liquidity supply. By reducing the liquidity supplied by a small amount, this generates two effects for profits via both cost savings and revenue generation. On the one hand, it leads to an increase in profits from cost savings when executing trades. On the other hand, the decrease in liquidity supplied causes the market price to increase at an increasing rate, because the investors are forced to trade more with the other dealers, who follow an increasing and convex liquidity cost process. These two effects complement each other and destroy any equilibrium. This further implies that investors have no incentives to divide their stock transactions among the potential market makers. As a consequence, market makers will necessarily engage in Bertrand price competition.

Our paper relates to two strands of the literature. First, it relates to the literature studying an investor's optimal trading strategy with liquidity risk (Cetin and Rogers, 2007; Vath et al., 2007; Chebbi and Soner, 2013; Pennanen, 2014), as well as equilibrium asset pricing models with liquidity risk (Jarrow, 2018a, 2019). Second, our paper relates to the literature on endogenous liquidity and market structure in competitive markets (Grossman and Miller, 1988; Bank et al., 2018). The dealers in our model are "traditional" market makers in the sense of Ho and Stoll (1981) and Kyle (1985) that do not hold inventory. Dealers in these alternative models face non-trivial inventory risks where embedding them into a general equilibrium asset pricing framework is more difficult (Garleanu et al., 2008; Muhle-Karbe and Webster, 2017).

An outline for this paper is as follows. Section 2 presents the model. Section 3 details the Bertrand price competition equilibrium where investors only seek service from one market maker. Section 4 conducts the equilibrium analysis for the cost competition game in which investors can approach multiple dealers. Section 5 discusses the dynamic incentives for dealers and concludes the paper.

2 The Model

This section presents the model. The starting point of our model is the liquidity cost process or supply curve, which is a building block for portfolio choice and asset market equilibrium theory with liquidity risk.[1] In such an economy, we are given a filtered probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0 \le t \le T}, \mathbb{P})$ where $\mathscr{F} = \mathscr{F}_T$ and T is a fixed time. Traded in the economy are a stock and a money market account (mma), whose return is normalized to be zero and thus serves as the numeraire. The market makers execute trades from individual investors on an exchange, and there exist K market makers. Meanwhile, there exist I investors who maximize their terminal wealth at time T by trading repeatedly on the exchange. The investors are indexed by $i \in \{1, \cdots, I\}$.

2.1 The Liquidity Cost Process

Following the literature (Cetin et al., 2004; Cetin and Rogers, 2007; Jarrow, 2018a), we define $S_t(x, \omega)$ to be the per share market price of stock when the trade size is $x \in \mathbb{R}$ and the state is $\omega \in \mathbb{R}$. This is sometimes called the "supply curve". We further assume that $S_t(x, \omega)$ is

[1]We refer interested readers to Jarrow (2017, 2018a) for a complete discussion.

$\mathscr{B}(\mathbb{R}) \bigotimes \mathscr{F}_t$ measurable. We define $S_t(0,\omega) \equiv S_t$ to be the market price for zero trades, which can be interpreted as the market price in a world without liquidity risk. [2]

Given the state ω, the liquidity cost process $\varphi_t(x,\omega)$ is defined as the cost of selling/buying x shares at time t, that is,

$$xS_t(x,\omega) = \varphi_t(x,\omega)S_t(0,\omega). \tag{1}$$

Following Cetin and Rogers (2007) and Jarrow (2018a), we have the following definition.

Definition 1 *(Well-behaved Liquidity Cost Processes): a liquidity cost process $\varphi_t(x,\omega)$ is well behaved iff: i) $\varphi_t(x,\omega)$ is $\mathscr{B}(\mathbb{R}) \bigotimes \mathscr{F}_t$ measurable for all t; and ii) $\varphi_t(x,\omega) : \mathbb{R} \to (-\infty,\infty]$ is strictly increasing and convex and differentiable in $x \in \mathbb{R}$ for all $t \in \{0,1,...,T\}$ and $\omega \in \Omega$ such that $\varphi_t(0,\omega) = 0$, $\varphi_t'(x,\omega) = 1$ a.s. $\mathbb{P}$.* [3]

As noted by Jarrow (2018a), this is a very general liquidity cost process. Two comments are in order. First, the monotonicity condition in conjunction with the condition that $\varphi_t(0,\omega) = 0$ implies that $\varphi_t(x,\omega) > 0$ for $x > 0$ and $\varphi_t(x,\omega) < 0$ for $x < 0$. Second, convexity is used to incorporate a nonlinearity in the liquidity cost. Note that, without liquidity risk, the liquidity cost process reduces to $\varphi_t(x,\omega) = x$ and thus $\varphi_t'(x,\omega) = 1$ for all x, which means that the transaction and marked-to-market price coincide. Henceforth, in conjunction with $\varphi_t'(x,\omega) = 1$, the convexity implies that the larger the quantity purchased, the larger the price paid per share, and vice versa.

Example 1 *A typical example used in the literature is the following stochastic liquidity cost process*

$$\varphi_t(x,\omega) = \frac{e^{\alpha_t(\omega)x} - 1}{\alpha_t(\omega)},$$

where $\alpha_t(\omega) \in \mathbb{R}_{++}$ is $\mathscr{F}_t$ measurable. Simple algebra verification shows that the stochastic liquidity cost process is well behaved.

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[2] The supply curve $S_t(x,\omega)$ is quite general and consistent with those processes generated by information effects or supply/demand imbalances in the market microstructure literature (Madhavan, 2000; Biais et al., 2005).

[3] In the literature, it is also assumed that $\varphi_t'(-\infty,\omega) = 0, \varphi_t'(\infty,\omega) = \infty$ and $S_t\tilde{\varphi}_t(-\frac{c}{S_t}) \in L^1(\mathbb{P})$ where $\tilde{\varphi}_t$ is the concave dual function define as $\tilde{\varphi}_t(w) = \inf_{x\in\mathbb{R}}\{\varphi_t(w) + wx\}$. Hence, our definition is weaker. We do not need this extra structure for our purposes.

2.2 The Market Maker's Profit

At each time t and state $\omega \in \Omega$, the market maker $k \in \{1, ..., K\}$ can execute a trade of size $x \in \mathbb{R}$ at the cost of $c_t^k(x, \omega) S_t$ dollars, where $c_t^k(x, \omega) : \{0, 1, ..., T\} \times \mathbb{R} \times \Omega \to \mathbb{R}$. This cost can be positive or negative. These costs reflect information asymmetries, search considerations, inventory risks, and fixed and variable expenses involved with running a market maker's business. For ease of presentation, we name this the trade execution costs. More importantly, this cost process is exogenous to the model, and we impose the following assumption on $c_t^k(x, \omega)$ for all of the market makers.

Assumption 1 *The cost processes $c_t^k(x, \omega)$ are convex for all $k \in \{1, \cdots, K\}$ and $t \in \{0, \cdots, T\}$.*[4]

We can now determine the market makers' profits. On each trade of size x executed, the trade generates cash into (out of) the market maker's position by $\varphi_t(x, \omega) S_t$, net the trade execution costs $c_t^k(x, \omega) S_t$. Given the stock trades $\Delta X_{t+1}^i : i \in \{1, \cdots, I\}$ submitted at time $t \in \{0, \cdots, T-1\}$, the net profit for dealer k is given by:[5]

$$\pi_t^k = \sum_{i=1}^{I} \left(\varphi_t^k(\Delta X_{t+1}^i, \omega) - c_t^k(\Delta X_{t+1}^i) \right) S_t. \tag{2}$$

We assume that the market maker's goal is to maximize her profits, that is, at each time t and state ω, the market maker $k \in \{1, \cdots, K\}$ maximizes (2). The appropriate equilibrium concept here is a Nash equilibrium in the trade execution function, i.e. the k-th market maker's strategy space consists all the possible liquidity cost functions $\varphi_t^k(x, \omega)$. In an equilibrium, fixing the equilibrium strategy taken by all the other market makers $j \neq k$, the k-th market maker's optimal strategy should maximize (2) for each $(x, \omega) \in \mathbb{R} \bigotimes \Omega$. The variant version of a Nash equilibrium adopted here can be traced back to the Cournot Nash equilibrium in the supply function as in Grossman (1981).

[4]Note that we do not assume symmetry on the execution cost processes and the convexity assumption can be relaxed.

[5]The trade submitted Δ_{t+1}^i is $\mathscr{F}_t$ predictable. In other words, it can only depend on the information contained in the filtration $\mathscr{F}_t$.

3 Bertrand Price Competition

For ease of presentation, we first present the Bertrand price competition game in which the investors only seek service from one market maker. Basically, this is the setup studied by Jarrow (2018a) and Jarrow (2019). There are two key assumptions here. First, the market makers are price-takers in the sense that they take as given the equilibrium price S_t. This transient price impact approach differentiates our model from those papers that model dynamic intermediation where in equilibrium, market makers face inventory risk in the aggregate level and try to maximize their terminal wealth (Bank et al., 2018; Kramkov et al., 2016). Second, the investors execute trades with only one market maker. This may arise when there exists market frictions preventing the investor from seeking trade executions with multiple dealers, or it may simply arise as an implication of equilibrium (as stated later in our model). Then, standard arguments show that a Bertrand price competition will naturally lead to the domination of cost-efficient market makers.

For ease of reference, denote the lowest trade execution cost market maker as $k_t^\star(x,\omega) = \text{argmin}\{c_t^k(x,\omega)\}$, and if there exist multiple $k_t^\star$ which achieve the lowest cost, then choose one of these arbitrarily. Furthermore, define

$$\varphi_t^\star(x,\omega) = \min_{k\neq k^\star}\{c_t^k(x,\omega)\},\ a.s.\ \mathbb{P}. \tag{3}$$

to be the next to trade execution cost among the market makers. Our previous observation is summarized in the following proposition.

Proposition 1 *(Bertrand Price Competition): there exists a Nash equilibrium such that all investors trade with the least cost of execution market maker, $k_t^\star(x,\omega)$, at the liquidity cost process given by $\varphi_t^\star(x,\omega)$, a.s., $\mathbb{P}$. All other market makers choose a liquidity cost process $\varphi_t^k(x,\omega) \geq c_t^k(x,\omega)$.*

The equilibrium verification in proposition 1 is very straightforward. First, note that the market makers never set a liquidity cost process below the cost process $c_t(x,\omega)$, otherwise she may earn a negative profit. Second, the market maker with the least cost of execution $k_t^\star(x,\omega)$ should not set the liquidity cost process larger than $\varphi_t^\star(x,\omega)$; otherwise, the market maker with the second-lowest cost can set a price at $\varphi_t^{k^\star} - \varepsilon$, where $\varepsilon > 0$ is sufficiently small. Third, note that in equilibrium all investors should trade with the cost efficient market maker $k_t^\star(x,\omega)$, even if there may exist other market maker(s) offering the same price. Otherwise, the market maker $k_t^\star(x,\omega)$ has an incentive to further lower the price by a small amount $\varepsilon > 0$.

Two comments are in order. First, price competition always chooses the market maker with the lowest cost, but the equilibrium liquidity cost process is determined by the *second-lowest* market maker's cost process, not always the *zero-profit* condition as stated in a standard Bertrand price competition game with identical dealers. Moreover, the cost-efficient maker is defined with respect to a state ω, a time t, and even a trade size $x \in \mathbb{R}$. Given proposition 1, any additional assumptions imposed on $c_t(x, \omega)$ will be reflected in the supply curve used in Cetin and Rogers (2007) and Jarrow (2018a).

4 A Cost Competition Game with Multiple Trades

This section presents a cost competition game with multiple trades. In other words, we remove the key assumption that each investor trades with only one market maker. Instead, investors can trade with multiple market makers and there are no frictions preventing investors from seeking service from multiple dealers. The main result below shows that there does not exist an equilibrium supporting both multiple market makers earning strictly positive profits and a well-behaved liquidity cost process (i.e. strictly increasing and convex). This implicitly implies that the Bertrand price competition structure is the natural equilibrium for our economy.

To facilitate the analysis, we stress an assumption not explicitly mentioned above. Conditional on the information filtration $\mathscr{F}_t$, market makers can infer each investor's trade demand ΔX^i_{t+1}. Basically, this is a symmetric information assumption. We use a variant of a Nash equilibrium in which a typical market maker's strategy space consists of all the possible liquidity cost processes $\{\varphi^k_t(x, \omega)\}$. This equilibrium concept is similar to the supply function equilibrium as in Grossman (1981) *per se*. Note that the cardinality of the trade demand/supply submitted, ΔX^i_{t+1} , is finite, but a typical market maker's strategy specifies the liquidity cost function charged for every $x \in \mathbb{R}$.

Consider the simplest case when there exist two identical market makers on the exchange (i.e., $K = 2$). The argument below can be easily extended to any integer $K > 2$. Next, we derive the equilibrium supply curve $\varphi^\star_t(x, \omega)$ and find conditions such that $\varphi^\star_t$ is well-behaved, that is, strictly increasing and convex. Note that given the price-taking assumption, we need to check that the market makers' incentive to follow the equilibrium liquidity cost process, given (i) the conjectured market equilibrium stock price process S_t, (ii) the exogenous trade execution cost $c_t(x, \omega)$, (iii) the conjecture about the investors' demand ΔX^i_{t+1} conditional on the information filtration $\mathscr{F}_t$ and that the other market makers follow the equilibrium strategy $\varphi_t^\star(x,\omega)$. To simplify notation, here we suppress the unnecessary super/subscripts when no confusion arises.

First, by the symmetry of the market makers and their trade execution cost structure, each investor will divide her demand among the two potential market makers evenly, as long as the equilibrium liquidity cost process $\varphi_t^\star(x)$ is indeed well-behaved. We will come back to this later.

Second, we need to check that market maker 1's incentive is not to deviate.[6] Given the stock trade demand/supply $\frac{\Delta X}{2}$ from a particular investor, the payoff from following the equilibrium strategy $\varphi_t^\star(x)$ is given by:

$$\pi(0) = \varphi_t^\star\left(\frac{\Delta X}{2},\omega\right)\frac{\Delta X}{2} - c_t\left(\frac{\Delta X}{2},\omega\right). \tag{4}$$

Now, consider a deviation for market maker 1 by switching to a new supply curve $x \mapsto \varphi_t^\star(x+2dx,\omega)$, where $dx > 0$ is sufficiently small. This is equivalent to a downward deviation for market maker 1 by reducing the supply of liquidity by a small amount dx. To see it, given these two asymmetrical supply curves $\varphi_t^\star(\cdot,\omega)$ and $\varphi_t^\star(\cdot+2dx,\omega)$, the investor with a trade size of Δx units will purchase a quantity of $\frac{\Delta x}{2} - dx$ units from market maker 1 and the remaining $\frac{\Delta x}{2} + dx$ units from market maker 2, leading to a trading price at $\varphi_t^\star(\frac{\Delta x}{2}+dx,\omega)$.[7]

Note that this will generate two effects in both revenues generation and cost savings. Specifically, the highest price that market maker 1 can now charge is given by $\varphi^\star(\frac{\Delta X}{2}+dx,\omega)$. This is the price charged by both market makers when the investor needs to shift a purchase of dx more units of the stock from market maker 1 to 2. On the other hand, the trade execution cost also decreases to $c_t(\frac{\Delta X}{2}-dx,\omega)$. In total, the payoff from shrinking the client base by dx is given by:

$$\pi(dx) = \varphi_t^\star\left(\frac{\Delta X}{2}+dx,\omega\right)\left(\frac{\Delta X}{2}-dx\right) - c_t\left(\frac{\Delta X}{2}-dx,\omega\right). \tag{5}$$

For ease of reference, also denote $x = \frac{\Delta X}{2}$, which should not cause confusion here. Then, the incentive compatibility condition, $\pi(0) \geq \pi(dx)$, reduces to:

$$\varphi_t^\star(x,\omega)x - c_t(x,\omega) \geq \varphi_t^\star(x+dx,\omega)(x-dx) - c_t(x-dx,\omega). \tag{6}$$

[6] The incentive not to deviate for market maker 2 holds by symmetry.

[7] We wound like to thank Neelanjan Datta to remind us to be explicit about the strategy space here.

Assuming differentiability, Taylor's expansion for the RHS above yields[8]

$$\begin{aligned}\varphi_t^\star(x+dx,\omega)(x-dx) - c_t(x-dx,\omega) &= \left(\varphi_t^\star(x) + (\varphi_t^\star)'(x)dx + \frac{1}{2}(\varphi_t^\star)''(x)(dx)^2\right)(x-dx) \\ &- \left(c_t(x) - c_t'(x)dx + \frac{1}{2}c_t''(x)(dx)^2\right) + o((dx)^3) \\ &= \varphi_t^\star(x)x - c_t(x) + D_1 dx + D_2(dx)^2 + o((dx)^3),\end{aligned}$$

where

$$D_1 = (\varphi_t^\star)'(x)x - \varphi_t^\star(x) + c_t'(x), \text{ and } D_2 = \frac{1}{2}(\varphi_t^\star)''(x)x - (\varphi_t^\star)'(x) - \frac{1}{2}c_t''(x).$$

Now, the optimality condition for equilibrium strategy requires $D_1 = 0$ and $D_2 \le 0$, that is,

$$\begin{aligned}&(F.O.C.) \qquad (\varphi_t^\star)'(x)x - \varphi_t^\star(x) + c_t'(x) = 0, \\ &(S.O.C.) \qquad \frac{1}{2}(\varphi_t^\star)''(x)x - (\varphi_t^\star)'(x) - \frac{1}{2}c_t''(x) \le 0;\end{aligned} \tag{7}$$

Differentiating over the F.O.C. condition in expression (7),

$$(\varphi_t^\star)''(x)x + c_t''(x) = 0,$$

implies the S.O.C. is further reduced to $-(\varphi_t^\star)'(x) - c_t''(x) \le 0$.

The following lemma characterizes the solution to the differential equation (7).

Lemma 1 *The general solution to the F.O.C. optimality condition is given by:*

$$\varphi_t^\star(x,\omega) = k(x,\omega)x, \tag{8}$$

where $k_t(x,\omega) = 1 + \lim_{x\to 0}\frac{c_t'(x,\omega)}{x} - \int_0^x \frac{c_t'(u,\omega)}{u^2}du$.

Proof. The general solution to F.O.C. is given by $\varphi_t^\star(x) = k * x$, where $k \ge 0$. In order to get the complete solution, set $k = k(x)$ and plug it into the optimality condition (1),

$$(k'(x)x + k)x - \varphi_t^\star(x) + c_t'(x) = 0,$$

[8]Note that a strictly increasing and convex function is almost differentiable everywhere. Thus, the differentiability assumption is not a very strong assumption.

Rearranging, yields

$$k'(x) = -\frac{c_t'(x)}{x^2},$$

After simple integration,

$$k(x) = c_0 - \int_0^x \frac{c_t'(u)}{u^2} du,$$

Imposing the boundary condition $\varphi_t^\star(0) = 0$ and $(\varphi_t^\star)'(0) = 1$ completes the calculation for c_0 and concludes the proof. ■

Remark 1 *When $c_t(x,\omega) = 0$, then $\varphi_t^\star(x,\omega) = x$, which reduces to the liquid market setting. Actually, it is not difficult to check this is indeed an equilibrium.*

With the aid of lemma 1, we come to the main result.

Theorem 1 *(A Non-trivial Supply Curve Equilibrium Does Not Exist): Under the convex cost process assumption 1, there does not exist a N.E. such that the equilibrium liquidity cost process is well behaved.*

Proof. With the aid of lemma 1, after simple algebra, we get

$$(\varphi_t^\star)'(x,\omega) = 1 + \lim_{x\to 0} \frac{c_t'(x,\omega)}{x} - \int_0^x \frac{c_t'(u,\omega)}{u^2} du - \frac{c_t'(x,\omega)}{x},$$

and

$$(\varphi_t^\star)''(x,\omega) = -\frac{c_t''(x,\omega)}{x}.$$

Henceforth, for general convex trade execution cost processes, the equilibrium liquidity cost process is concave, whenever the equilibrium exists. This in turn implies that, the investors have no incentive to trade with multiple market makers under a concave cost process. ■

The main result, Theorem 1, has important implications for the market microstructure of an economy with a liquidity cost process. It negates the existence of any equilibrium which features a well-behaved liquidity cost process and a non-trivial market structure such that multiple market makers coexist with strictly positive profits. This is not to say that a well-behaved liquidity cost process does not exist, nor can the market maker not earn strictly positive profits. Instead, it says that these two cannot coexist in the sense of a Cournot equilibrium. This lends support to the equilibrium construct as in the Bertrand price competition case.

The intuition behind this result is straightforward. Given a convex trade execution cost process, if there exists an equilibrium which features a well-behaved liquidity cost process, then the market makers have an incentive to reduce the liquidity supplied. By reducing the liquidity supplied by a small amount, it generates two positive effects for profits via both a cost savings to execute the trade and revenue generation. The decrease in the liquidity supplied further causes the market price to increase at an increasing rate, because investors are forced to trade more liquidity with the other dealers, who use an increasing and convex liquidity cost process. These two effects complement each other and thus destroys the equilibrium.

5 Discussion and Conclusion

According to Butz and Oomen (2019), market makers provide liquidity to investors in two ways. One, they can warehouse inventory and manage the inventory risk position. Two, they can also externalize the trade by hedging the risk in the open market. Dealers in practice are a mix of both internalizers and externalizers. Considering this dichotomy, our model falls into the category of analyzing dealers who are pure externalizers similar to the traditional market makers in Ho and Stoll (1981).

Once we introduce the role of dealers as internalizers, the market makers face dynamic incentives when absorbing the liquidity risk. Specifically, the market makers are now concerned about gains/losses from trading with investors on the exchange, the efficient dissipation of inventory risks, and the maximization of terminal wealth. For example, Bank et al. (2018) extends the classical model Grossman and Miller (1988) to a dynamic partial equilibrium setting, while in Kramkov et al. (2016) these dynamic incentives have permanent effects on equilibrium stock price.

A potential challenge for the dynamic approach lies in the fact that it is a partial equilibrium analysis and thus difficult to interpret the asset pricing implications from a general equilibrium viewpoint. To be precise, the market makers can participate and hold assets by bearing inventory risk. The ongoing inventory risks across their lifetime differ from the preference structure of typical investors, which in turn destroys the existence of a representative trader and thus the representative trader equilibrium as in Jarrow (2018b).

To conclude, this paper endogenizes the liquidity cost processes in a non-cooperative market maker competition game. We show there does not exist an equilibrium supporting both multiple dealers earning positive profits and a well-behaved liquidity cost process. This helps explain why Bertrand price competition is a good approximation for the liquidity supply markets employed in the existing literature. For future research, it is still an open question how to embed the dynamic incentives mentioned above in a general equilibrium framework to get a well-behaved liquidity cost process as stated in Jarrow (2018a).

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